Effect of thickness on the compressive performance of ballistically impacted carbon fibre reinforced plastic (CFRP) laminates

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The response of structural elements under impact conditions is a particularly important consideration in the design of components made from composite materials. The understanding of this response includes both the impact behaviour and the influence of some design parameters and material properties. Thus, the dependence of the residual compressive strength of ballistically impacted carbon fibre reinforced plastic (CFRP) laminates on their thickness has been examined. A previously verified model developed by the authors, has been applied resulting in rather interesting findings about the effect of the thickness on the sensitivity of a laminate to impact. The model takes into account the number of plies, the impact energy and the stacking sequence. Experimental results derived from the literature have been used for the verification of the model and a close agreement between theoretical predictions and experimental results was found. Also, it can be concluded that the present work helps to optimize laminate impact behaviour by varying the laminate thickness.

Nomenclature

a	energy absorption capacity coefficient defined from a nomograph
$[D_{ij}]_0$	flexural stiffness matrix of the unimpacted material, (Nm)
$[D_{ij}]_r$	flexural stiffness matrix of the impacted material, (Nm)
D_{xx}	flexural stiffness in the longitudinal direction x as shown from the natural axis system of the laminate, (Nm)
M	bending stiffness mismatching coefficient
m	ratio of summations in the proposed model
n	total number of laminae in a laminate composite
$[Q_{ij}]$	stiffness matrix of a lamina, (Nm^{-2})
σ_0	compressive strength of the unimpacted material, (MPa)
σ_r	residual compressive strength of the impacted material, (MPa)
U	impact energy, (J)
U_0	impact energy threshold, (J)
x	fraction of the $\pm 45°$ plies in the laminate, (%)
h_{tot}	total laminate thickness, (mm)
t	lamina thickness, (mm)
z_κ	distance of the κ-lamina from the middle plane of the laminate, (m)
θ_κ	fibre orientation in the κ-lamina between two adjacent laminae
$\theta_{\kappa+1}$	fibre orientation in the (κ + 1)-lamina between two adjacent laminae

1. Introduction

The damage to laminated composites caused by low velocity impact has been extensively studied [1–8]. The nature of the impact damage in fibre reinforced composite laminates ranges from surface damage and subsurface damage to complete penetration, depending on the impact loading conditions. Generally, under low velocity impact, damage only occurs in certain plies of a laminate, resulting in so-called part through-the thickness damage [9, 10]. This kind of damage consists mainly of matrix-controlled failure modes, transverse cracks and delamination.

Improvements are being made continually in composite materials in order to improve their impact performance and reduce the amount of damage, so that more efficient use can be made of composites in many technological areas. The strength of carbon fibres is now of the order of 6 GPa with failure strains in excess of 2%, thus the strain energy to failure has been significantly increased. Improvements in the toughness of the matrix, by either toughening of the thermosets or by the use of thermoplastics, so as to reduce the damage area and to resist delamination growth on subsequent compression loading have been reported. Effort is also being expended to optimise fibre surface treatments for specific combinations of fibres and resins [11].

Although the impact phenomenon itself is one of the most important and complex dynamic loadings, the materials response to this type of loading is closely related to its structure. Thus, excluding all the

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improvements mentioned above, there are still a great number of design parameters that strongly affect the impact behaviour of composite materials. Investigations into this behaviour have shown that parameters such as impact energy, stacking sequence and geometry of the tested specimens, play a significant role in the optimum design of composite structures.

A general approach for the modelling of the loss of compressive strength of composite laminates due to impact damage has been previously developed [12]. According to this model, parameters such as the stacking sequence of the laminate, the energy absorption capacity of the material under consideration, and the impact energy are related to the normalized residual compressive strength (σ_r/σ_0) after impact.

It was experimentally found that the damage threshold for an impacted laminate is strongly dependent on the thickness [13]. With increasing laminate thickness, h_{tot}, the damage threshold moves to higher impact energies. This results in the conclusion that major changes in energy absorption or damage extent can happen for thicker laminates. An increase of the impact energy threshold, U_0, with a parallel limitation of the damage extent shows that the toughness of the laminate is significantly affected as the thickness increases. In the same work, it was also reported that the residual compressive strength after impact is dependent on the laminate thickness. Thus, considering the damage size effect it appears that thick laminates are less susceptible to impact damages than are thin ones.

In the present work, a model previously developed and presented elsewhere by the authors [12] will be used in order to study the laminate thickness effect on the compressive performance of ballistically impacted carbon fibre reinforced plastic (CFRP) laminates. Following this, comparisons between experimental results found in the literature and theoretical predictions based on the model will be made.

2. Theoretical background

The modes of failure in a laminated plate produced by impact of a hard object are quite complex. In general, there are three major modes of failure: (i) matrix cracking in the lamina, (ii) transverse shear and bending cracks, and (iii) delamination. For a [0°/90°/0°] laminate, the typical damage pattern due to impact is shown in Fig. 1(a and b) [14].

Amongst all these modes of impact damage, delamination has the most detrimental effect on laminate stiffness and strength, and thus has received a considerable amount of attention [15]. Thus, allowing for the fact that delamination results in the degradation of most of the mechanical properties such as stiffness and strength, the proposed model developed in reference [12], is based on the assumption that the degradation of the flexural stiffness term, D_{xx}, is related to the residual compressive strength as follows:

$$\frac{\sigma_r}{\sigma_0} = \frac{D_{xx,r}}{D_{xx,0}} \tag{1}$$

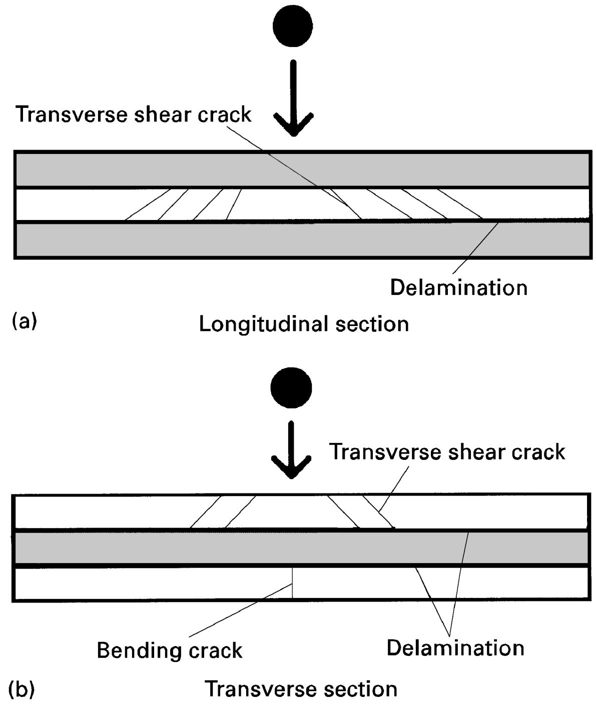

Figure 1 Typical damage pattern due to impact. (a) Longitudinal section (b) Transverse section [14].

where, σ_r represents the residual compressive strength of the impacted laminate, σ_0, is the strength of the unimpacted material, $D_{xx,0}$ is the already known, from classical lamination theory, flexural stiffness matrix term of the unimpacted laminate and finally, $D_{xx,r}$, is the respective flexural stiffness matrix term of the impacted material.

Through the analysis presented in reference [12] the characteristic form of the proposed model is given by the following expression:

$$\frac{\sigma_r}{\sigma_0} = 4m\,\frac{10\alpha + 1}{U^{\alpha}} \tag{2}$$

where U is the impact energy, α is an energy absorption capacity factor and m is defined as follows:

$$m = \frac{\sum_{\kappa=1}^{n} (\overline{M_\kappa})_0 [Q_{xx,\kappa}(z_\kappa^3 - z_{\kappa-1}^3)]}{\sum_{\kappa=1}^{n} [Q_{xx,\kappa}(z_\kappa^3 - z_{\kappa-1}^3)]} \tag{3}$$

where $(\overline{M_\kappa})_0$ is the mean value for the bending stiffness mismatching coefficient of the κ-lamina [16], $Q_{xx,\kappa}$ is the x-direction bending stiffness matrix term of the κ-lamina, z_κ is the distance of the κ-lamina from the middle plane of the laminate and n is the total number of layers in the laminate. More precisely, the mean value of $(\overline{M_\kappa})_0$ shown in the above expression, is defined as follows:

$$(\overline{M_\kappa})_0 = \frac{(M_{\kappa-1,\kappa})_0 + (M_{\kappa,\kappa+1})_0}{2} \tag{4}$$

where $(\overline{M_\kappa})_0$ refers to κ-lamina and $M_{\kappa-1,\kappa}$ and $M_{\kappa,\kappa+1}$ refer to the interfaces of the adjacent layers ($\kappa - 1$), κ and ($\kappa + 1$) shown in Fig. 2.

Based on experimental results [12], a linear variation of α with the fraction of the $\pm 45°$ layers in the laminate was found and this is shown in Fig. 3. An

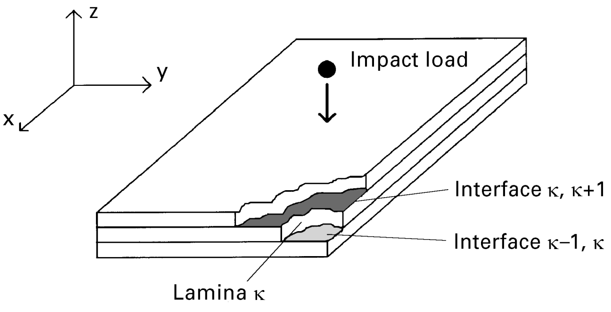

Figure 2 Description of the laminate configuration used in the proposed model.

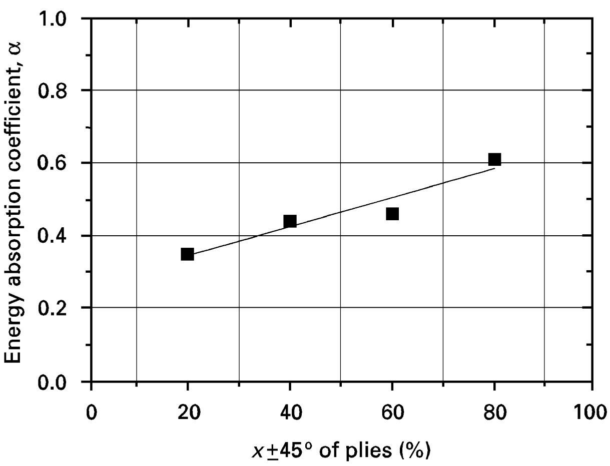

Figure 3 Variation of the energy absorption coefficient (×) as a function of x% of ±45° plies in a laminate. Key: (■) Experimental values; (——) Linear fit.

analysis of the model as presented in reference [12], is given in the Appendix.

3. Materials

The experimental results used in this work were taken from the literature [17] and all test specimens were manufactured from unidirectional tapes of the type Ciba 6376/HTA. Four types of lay-up were considered. The thickness of each laminate was varying by increasing the number of the layers. Thus, a laminate lay-up $(+45°, 0°, -45°, 90°, -45°, 0°, +45°)_\mu$ was used, where $\mu = 2, 3, 4, 6$. Impacts were produced by means of an air-gun system and the specimens dimensions were $250 \times 110 \times 5.25$ mm (maximum value of thickness for $\mu = 6$). The results for the residual compressive strength after impact were used in the present investigation in order to verify predictions derived from our model. These predictions appreciate the effect of the laminate thickness on the residual compressive strength after subjecting the specimens to 32 and 48 J impact energies.

4. Results and discussion

According to the previous discussion, the major limiting design factor is the susceptibility of the material to impact damage in the form of multiple delaminations through the thickness. These internal delaminations, which are produced by a dynamic impact loading applied perpendicular to a laminate, result in a degradation of the compressive strength of the plate. This strength reduction is usually attributed to the buckling of the sublaminates formed by the internal delaminations under inplane compressive loads. This buckling of the sublaminates will trigger unstable delamination growth which in turn will lead to the premature failure of the laminated plate [18].

Also, internal stress waves and local out-of-plane deformations may initiate delamination at interfaces where there is a major change in the angle between plies. Earlier work directed towards characterizing the damage mode of delamination and finding ways to reduce the delamination area and to increase the toughness of the material, has shown that the thickness of the laminate affects strongly the residual compressive behaviour of ballistically impacted CFRP laminates [18].

In the present work, in order to compare the theoretical predictions derived by the proposed model with the respective experimental findings taken from reference [17], the α-value needed for the application of the model was determined from the nomograph shown in Fig. 3. Thus, according to this graph, $\alpha = 0.48$ when the fraction of the layers with $\pm 45°$ fibre orientation is equal to $x = 57\%$. Next, by varying the value of μ, m was calculated as a function of the total laminate thickness, h_{tot}, and the results are shown in Table I.

The variation of the experimentally found residual compressive strength versus thickness of the CFRP laminate, is shown in Table II. The respective graphical representation showing the effect of laminate thickness on the residual compressive strength for two different impact energies is given in Fig. 4. In this figure it becomes clear that as the thickness increases up to 5 mm, the residual compressive strength, σ_r, also increases. Taking into account that the purpose of the

TABLE I Values of m parameter for different lay-ups

μ	1	2	3	4	6
h_{tot}	0.875	1.750	2.625	3.500	5.250
m	0.200	0.123	0.087	0.067	0.046

TABLE II Effect of thickness: test results and theoretical predictions for a laminate lay-up: $(+45°, 0°, -45°, 90°, -45°, 0°, +45°)_\mu$ with $\mu = 2, 3, 4, 6$

Thickness h_{tot} (mm)	σ_0 (MPa)	Experimental values of σ_r (MPa)	Predicted values of σ_r (MPa)
Impact energy $U_1 = 32$ J			
1.750	235	132	127.38
2.625	441	165	168.64
3.500	741	215	218.22
5.250	1106	215	224.31
Impact energy $U_2 = 48$ J			
1.750	235	93.5	104.85
2.625	441	132.0	138.81
3.500	741	173.0	179.63
5.250	1106	175.0	184.64

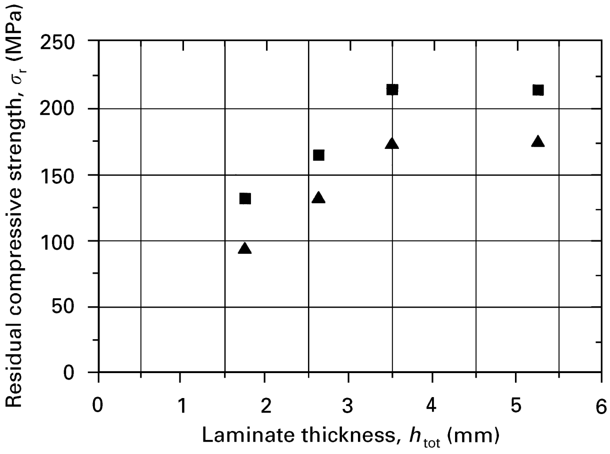

Figure 4 Effect of laminate thickness on the residual compressive strength after impact with energies; (■) $U_1 = 32$ J and (▲) $U_2 = 48$ J.

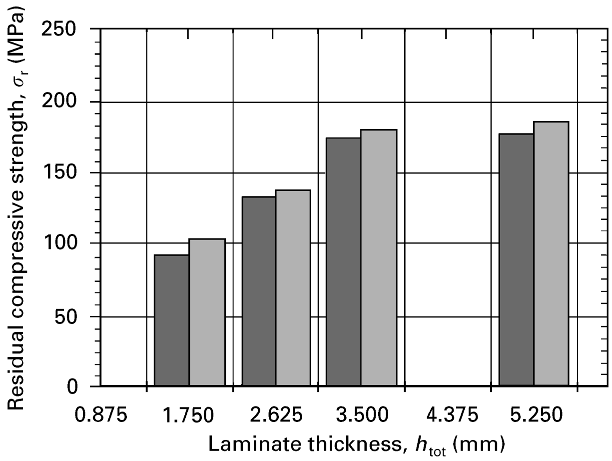

Figure 6 Residual strength increase as a function of laminate thickness. Impact energy $U_2 = 48$ J. Key (■) Test results, (□) Predicted values.

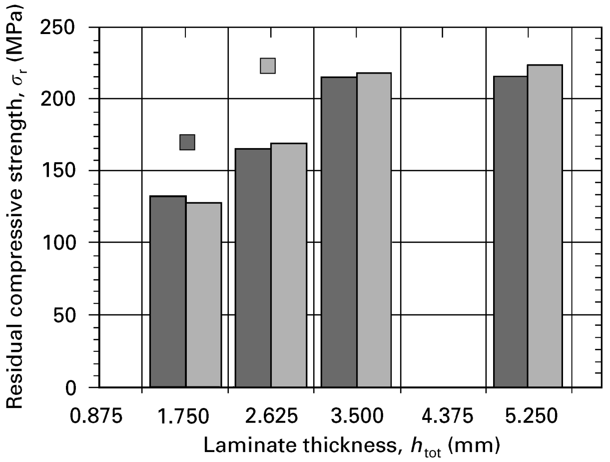

Figure 5 Residual strength increase as a function of laminate thickness. Impact energy $U_1 = 32$ J. Key (■) Test results, (□) Predicted values.

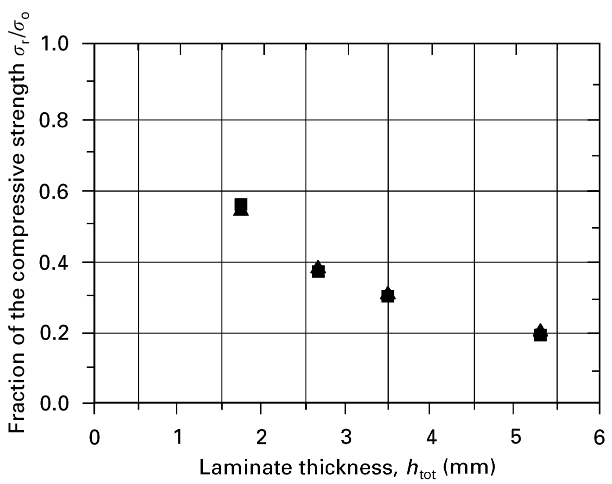

Figure 7 Variation of the fraction of the compressive strength (σ_r/σ_0) as a function of laminate thickness. Impact energy $U_1 = 32$ J. Key (■) Experimental results, (▲) Predicted values.

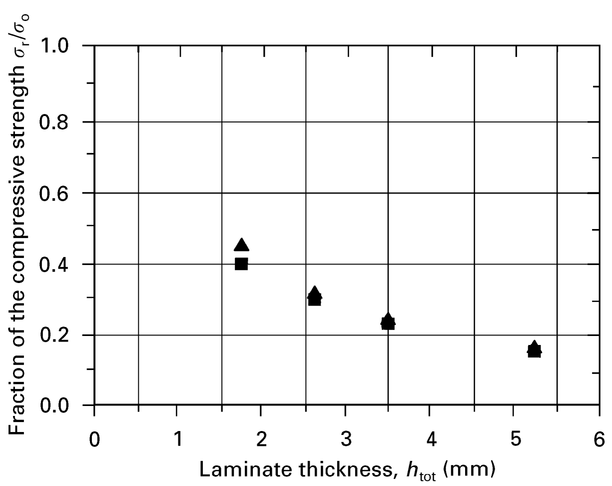

Figure 8 Variation of the fraction of the compressive strength (σ_r/σ_0) as a function of laminate thickness. Impact energy $U_2 = 48$ J. Key (■) Experimental results, (▲) Predicted values.

present study is to correlate the variation of laminate thickness and the residual compressive performance of the laminate, the predictions of the proposed model were used for the verification of the test results given in Table II.

Thus, σ_r-values were derived by using the already calculated m and α values. A very good agreement between the theoretical predictions and the experimental values was observed and this is better shown in Table II.

A graphical representation of the above comparison is shown in Figs. 5 and 6. In both diagrams an increase of the residual compressive strength, σ_r, versus laminate thickness, h_{tot}, is observed while the agreement between experimental values and theoretical predictions derived by the present model is clearly shown.

Next, if the normalized residual compressive strength (σ_r/σ_0) is plotted against h_{tot}, a decrease of the former with increasing h_{tot} is observed and this type of variation is shown in Figs. 7 and 8. This is due to the fact that as the thickness, h_{tot}, increases the initial compressive strength of the material, σ_0, also increases. However, the rate of increase of σ_0 is much higher than the respective rate of increase of the residual compressive strength of the material, σ_r. This type of behaviour can be explained by means of the toughness concept. More precisely, taking into account that the toughness is a measure of the ability of the

TABLE III Theoretical predictions for the compressive strength reduction after impact. Impact energies, $U_1 = 32$ J and $U_2 = 48$ J

Laminate lay-up: $(+45°, 0°, -45°, 90°, -45°, 0°, +45°)_\mu$ with μ: 1, 2, 3, 4, 6.

Layer thickness, $t = 0.125$ mm			
h_{tot}	1.750	3.500	5.250
m	0.123	0.067	0.046
($U_1 = 32$ J)			
σ_r/σ_0	0.542	0.294	0.202
($U_2 = 48$ J)			
σ_r/σ_0	0.446	0.242	0.166
Layer thickness, $t = 0.25$ mm			
h_{tot}	1.750	3.500	5.250
m	0.200	0.123	0.087
($U_1 = 32$ J)			
σ_r/σ_0	0.881	0.542	0.382
($U_2 = 48$ J)			
σ_r/σ_0	0.725	0.446	0.314

material to absorb released strain energy non-catastrophically by plastic deformation, shear cracking or delamination and to reduce stress concentration, the appreciation of this material property is one of the basic purposes in many research areas.

As a result, the impacted specimen is characterized by a reduced toughness when compared to the unimpacted specimen. Hence, the previously discussed difference in the rate of increase of the residual compressive strengths with laminate thickness is fully expected.

It is very interesting to note that, for impact loading conditions, an increase in the laminate thickness produced by an increase in each layer thickness, t, and not by changing the number of layers, n, results in higher values of the residual compressive strength. Of course, this increase becomes more obvious when comparing two laminates with the same total thickness. Thus, the above statement can be verified by means of the proposed model when applied to a laminate lay-up $(+45°, 0°, -45°, 90°, -45°, 0°, +45°)$. This is clearly shown in Table III, where the theoretical predictions obtained from our model are listed, while their graphical representation is shown in Figs. 9 and 10. So, for a laminate thickness equal to 3.5 mm, it is obvious that a lay-up $(+45°, 0°, -45°, 90°, -45°, 0°, +45°)_s$ with $t = 0.250$ mm gives a higher value for the fraction of the compressive strength than a lay-up $(+45°, 0°, -45°, 90°, -45°, 0°, +45°)_{2s}$ with $t = 0.125$ mm. The same behaviour is also observed for the other two values of h_{tot}.

This type of behaviour was expected since the possibility of interfacial weakening is reduced as the number of interfaces decreases. In this case, delamination, which plays a critical role in affecting the compressive behaviour since it may cause localized buckling, seems to be avoided.

It is very important also to note that the proposed model closely predicts the fraction of the residual compressive strength, σ_r, as well as the initial compressive strength, σ_0. This is due to the fact that the variation of the laminate thickness and the number of layers results in a change of the material properties

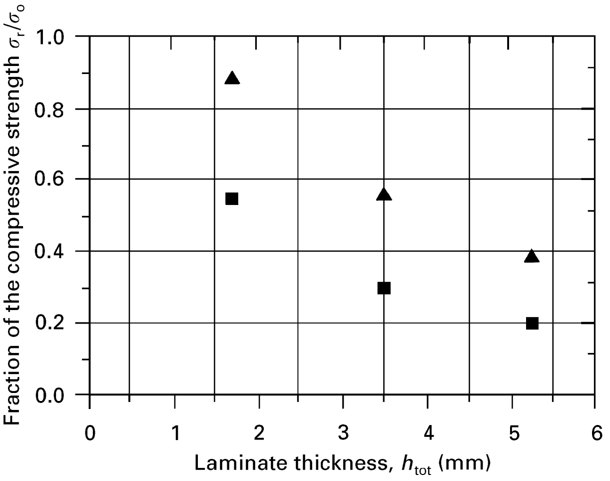

Figure 9 Variation of the fraction of the compressive strength (σ_r/σ_0) as a function of the laminate thickness, h_{tot}, at values of the layer thickness, t. Key (■) 0.125 mm and (▲) 0.250 mm. Impact energy $U_1 = 32$ J.

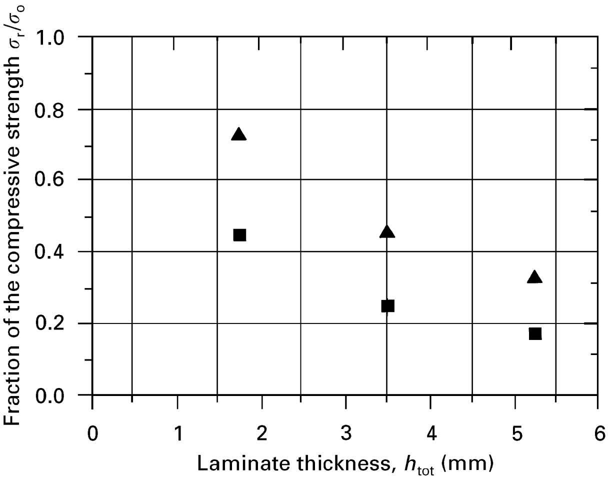

Figure 10 Variation of the fraction of the compressive strength (σ_r/σ_0) as a function of laminate thickness, h_{tot}, at values of the layer thickness, t. Key (■) 0.125 mm and (▲) 0.250 mm. Impact energy $U_2 = 48$ J.

TABLE IV Calculating values of $D_{xx,r}$ and $D_{xx,0}$ for different values of laminate thickness

n	h_{tot}	$D_{xx,0}$	$D_{xx,r}$
Layer thickness, $t = 0.125$ mm			
7	0.875	4.018	0.805
14	1.750	30.445	3.754
21	2.625	101.686	8.876
28	3.500	240.148	16.170
42	5.250	808.368	37.278
Layer thickness, $t = 0.25$ mm			
7	1.750	32.151	6.444
14	3.500	243.560	30.035
21	5.250	813.486	71.009
28	7.000	1921.190	129.365

[19]. As previously mentioned the proposed model is based on the degradation of the material properties which are expressed by the variation of flexural stiffness terms. From classical lamination theory, the flexural stiffness term $D_{xx,0}$, depends on the stacking sequence and the laminate thickness. Thus, calculating different values of $D_{xx,0}$ and $D_{xx,r}$ for different values of

laminate thickness, the reason why it is more preferable to increase the total laminate thickness with an increase in each layer thickness rather than increasing the number of layers can be explained. This is clearly shown in Table IV.

5. Conclusions

In the present work, a theoretical model previously developed by the authors for the prediction of the residual strength after impact of CFRP laminates has been used in order to study the thickness effect on the compressive performance of ballistically impacted CFRP laminates. The theoretical predictions were compared with experimental findings taken from the literature. In all cases, a good agreement was observed.

The results clearly demonstrate that the compressive performance of ballistically impacted laminates depends strongly on the variation of laminate thickness. The predicted values for the residual compressive strength of CFRP laminates and for a wide range of laminate thicknesses showed that the damage energy threshold increases with increasing the total laminate thickness and this in turn leads to higher values of the residual compressive strength. The compressive performance can be further improved if the increase in laminate thickness results from an increase of the individual laminae thickness and not by increasing the number of layers used in the laminate. Both, the model as well as the results of the present study can be used as a design tool for optimizing carbon fibre laminate architecture.

Appendix

The analysis of the proposed model [12], was based on the assumption that the compressive strength degradation of a laminated plate containing multiple interlaminar cracks and delaminated areas as a result of low velocity impact is related to the flexural stiffness degradation as follows:

$$\frac{\sigma_r}{\sigma_0} = \frac{D_{xx,r}}{D_{xx,0}} \tag{A1}$$

When a laminated plate is subjected to low velocity impact, the interlaminar damage that may be caused can be expressed by the mismatching with respect to the bending stiffness between each pair of adjacent laminate layers. The so-called bending stiffness mismatching coefficient, $M_{\kappa,\kappa+1}$, according to reference [16], expresses the difference in bending stiffness between two adjacent laminae and is defined as follows:

$$(M_{\kappa,\kappa+1})_0 = \frac{D_{xx,0}(\theta_\kappa) - D_{xx,0}(\theta_{\kappa+1})}{D_{xx,0}(0^\circ) - D_{xx,0}(90^\circ)} \tag{A2}$$

In order to define the relation between impact damage and impact energy, it was assumed that the extent of damage (which is qualitatively expressed by the total delaminated area size and quantitatively by the flexural stiffness matrix term, $D_{xx,r}$, of the impacted material), is expressed as follows:

$$D_{xx,r} = (\overline{M_\kappa})_0 \frac{d}{U^\alpha} D_{xx,0} \Rightarrow D_{xx,r} = \frac{1}{3}\frac{d}{U^\alpha}\sum_{\kappa=1}^{n}(\overline{M_\kappa})_0 \times [Q_{xx,\kappa}(z_\kappa^3 - z_{\kappa-1}^3)] \tag{A3}$$

where $(\overline{M_\kappa})_0$ is the mean value for the bending stiffness mismatching coefficient of the κ-lamina as shown in Equation 4 of the main text, and d is a parameter which depends on the material properties and test conditions.

So, by combining Equations 4, A1 and A3 the following relation can be derived:

$$\frac{\sigma_r}{\sigma_0} = \frac{\frac{d}{U^\alpha}\sum_{\kappa=1}^{n}(\overline{M_\kappa})_0[Q_{xx,\kappa}(z_\kappa^3 - z_{\kappa-1}^3)]}{\sum_{\kappa=1}^{n}[Q_{xx,\kappa}(z_\kappa^3 - z_{\kappa-1}^3)]} = \frac{D_{xx,r}}{D_{xx,0}} = m\frac{d}{U^\alpha} \tag{A4}$$

where m is the ratio of summations. Then, Equation A4 can also be written in the following form,

$$\log(\sigma_r/\sigma_0) = \log(md) - \alpha\log(U) \tag{A5}$$

and through the experimental analysis presented in reference [12] the model takes the characteristic form shown in Equation 2 of the main text.

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